Lecture 19: Digital Modulation

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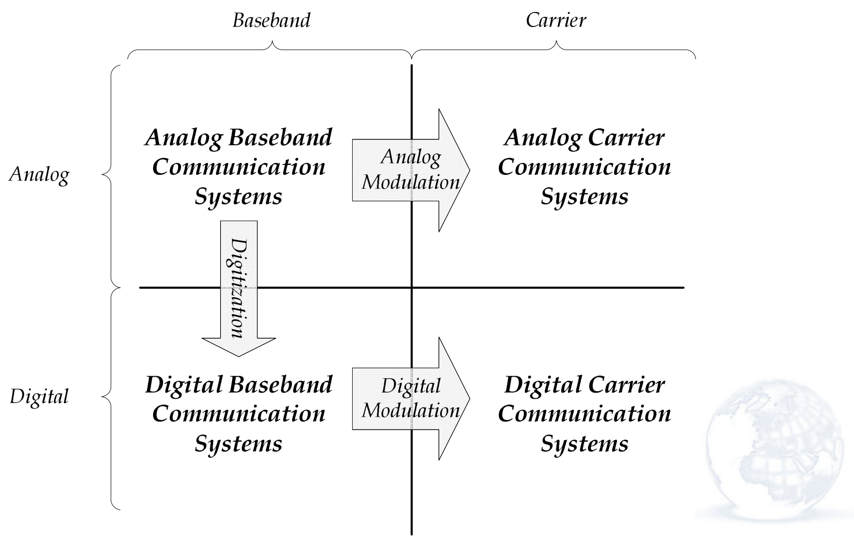
EE421: Communications I. For more information read Chapters 7 & 10 in your textbook or visit http://wikipedia.org/.

The *Last Piece* of the Puzzle!



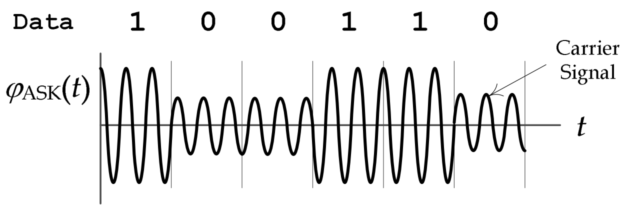
Digital Modulation

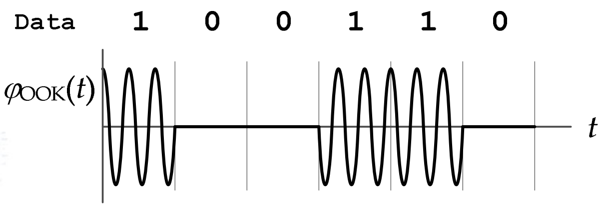
- *Four* main modulation techniques:
  - Amplitude-Shift Keying (**ASK**).
  - Frequency-Shift Keying (**FSK**).
  - Phase-Shift Keying (**PSK**).
  - Quadrature Amplitude Modulation (**QAM**).
- PSK and QAM are the most popular nowadays because of their *smaller* bandwidths.
- PSK and QAM require synchronous detection, which is easier nowadays (PLLs).

Analog vs. *Digital* Modulation

- $m(t)$ *is* Polar NRZ + AM = **ASK**
- $m(t)$ *is* Polar NRZ + FM = **FSK**
- $m(t)$ *is* Polar NRZ + PM = **BPSK**
- $m(t)$ *is* Q-ary NRZ + PM = **QPSK**
- $m(t)$ *is* M-ary NRZ + PM = **M-PSK**
- $m(t)$ *is* M-ary NRZ + QAM = **QAM**
- $m(t)$ *is* M-ary NRZ + AM = **M-ASK**

ASK and OOK





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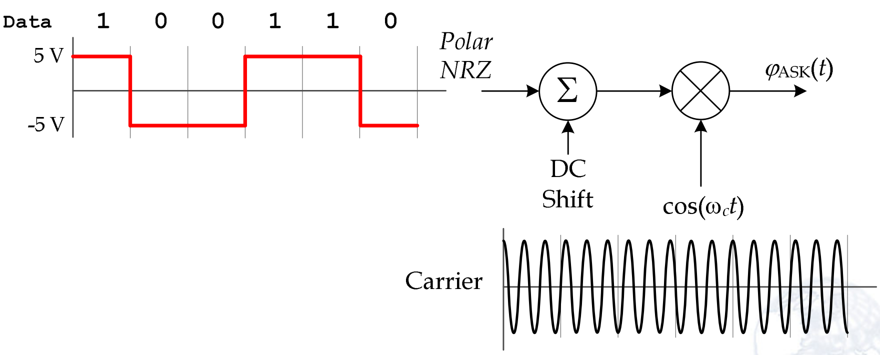
ASK *is* AM



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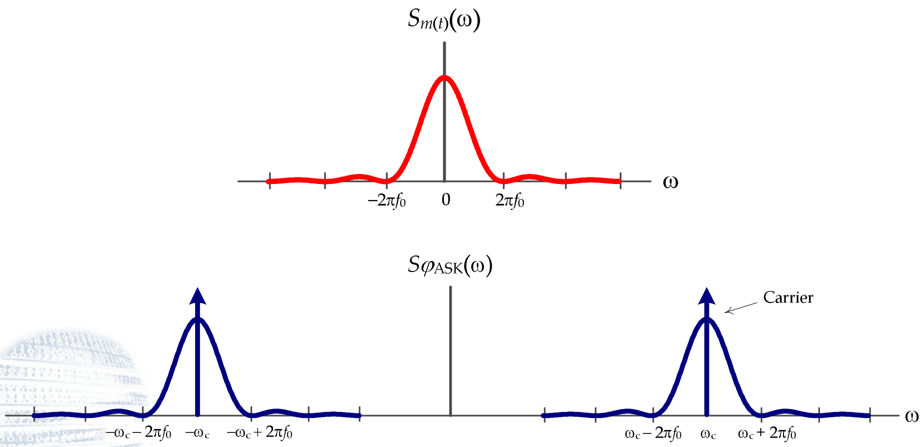
PSD of an ASK Signal



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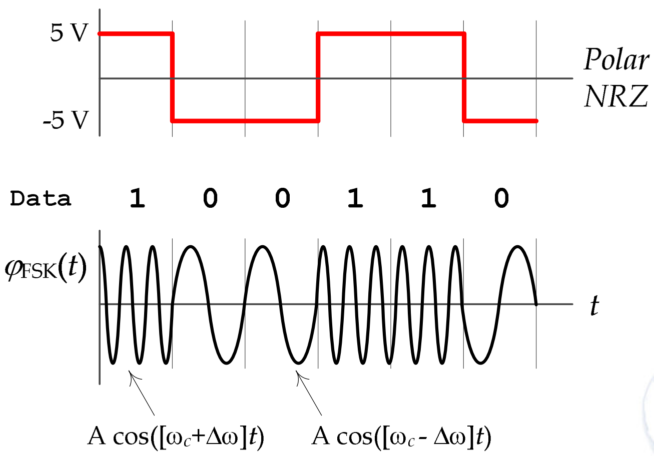
FSK



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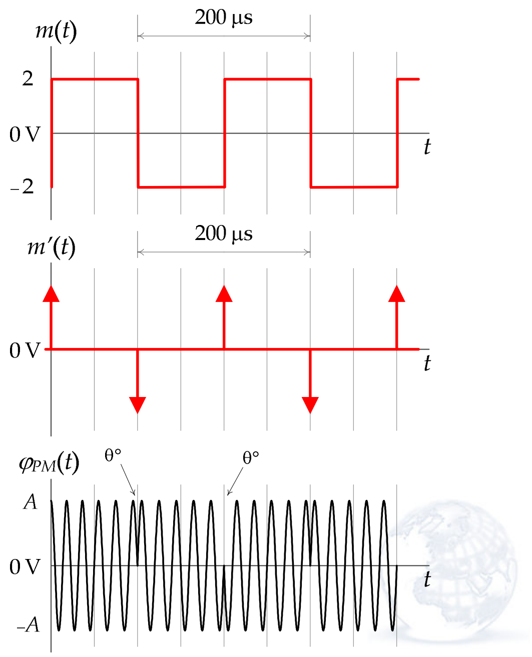
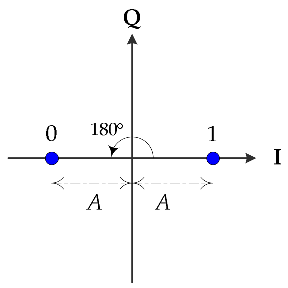
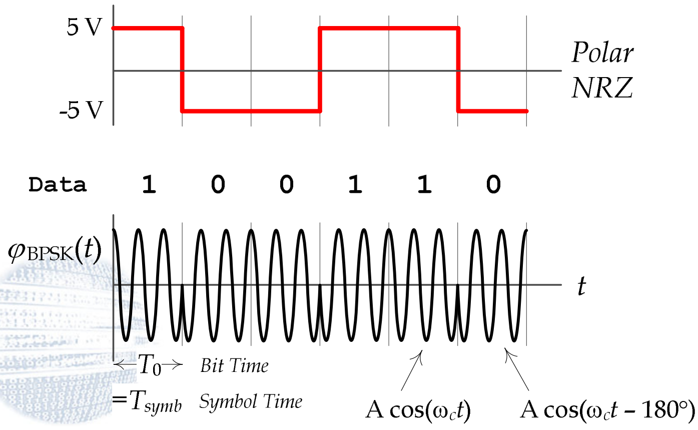
BPSK = Polar NRZ + PM

Constellation Diagram is a shortcut for *t*-domain

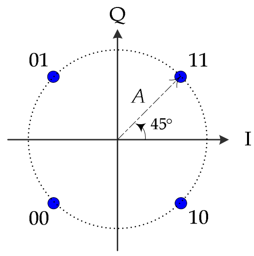


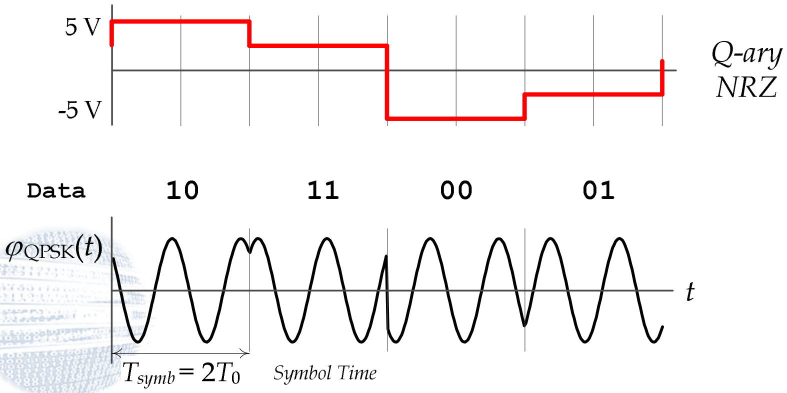
BPSK





QPSK

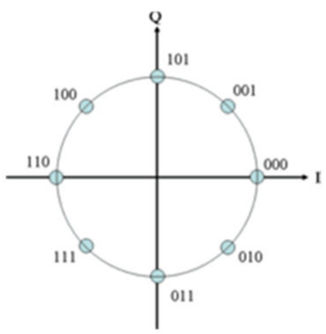




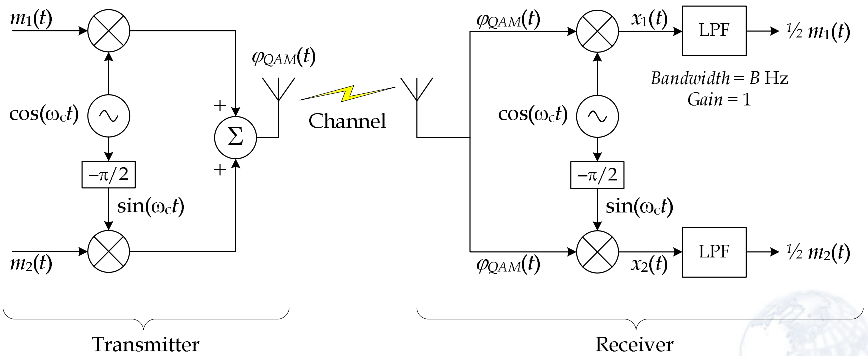
8-PSK

Example: Find the bandwidth of an 8-PSK modulated signal if the data bit rate is 100 kbit/s.

Solution: For 8-PSK, Bandwidth = 2B = 2×Baud Rate = $2 \times \frac{100\,kbps}{\log_2(8)} = 2 \times \frac{100\,kbps}{3\ bits/symb} =$ 66.67 kHz.



QAM: *Analog*



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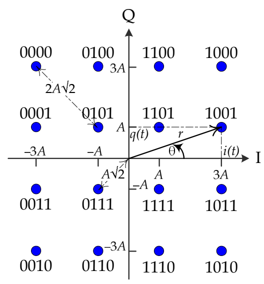
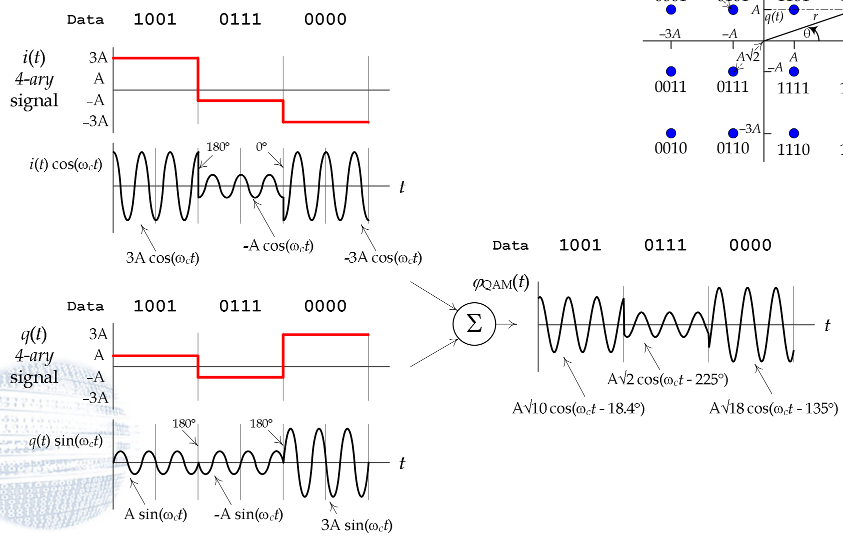
QAM: *Digital*





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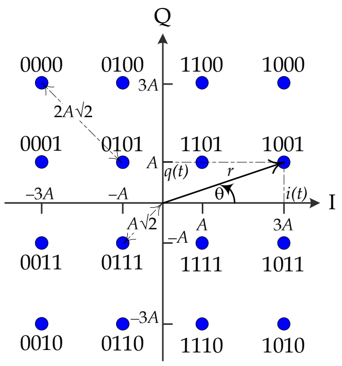
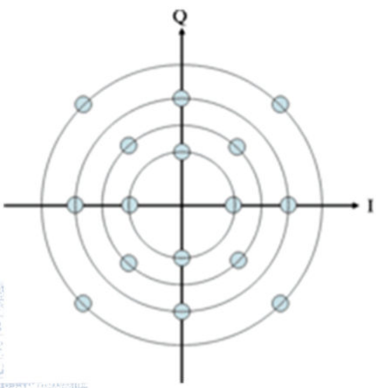
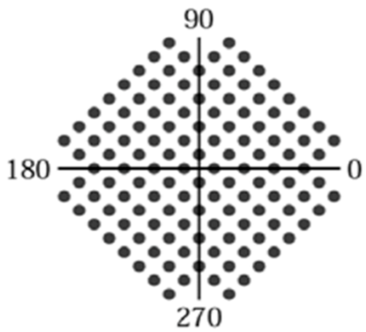
16-QAM Constellation Diagram



Example: Find the bandwidth of an 16-QAM modulated signal if the data bit rate is 8 Mbit/s.
Solution: For 16-QAM, Bandwidth = 2×Baud Rate = $2 \times \frac{8Mbps}{\log_2(16)} = 2 \times \frac{8Mbps}{4\ bits/symb}$ = 4 MHz.

Many QAM Constellations

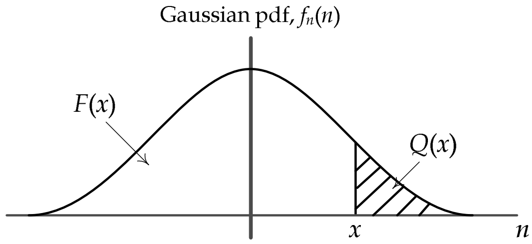




AWGN Noise

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

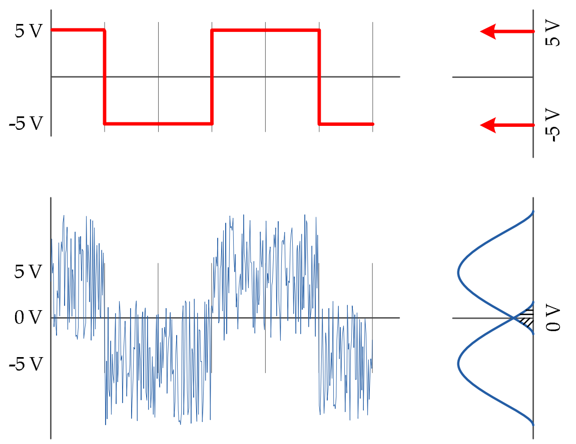
$$Q(x) = 1 - F(x) = 1 - \int_{-\infty}^{x} f(\alpha) d\alpha = \int_{x}^{\infty} f(\alpha) d\alpha = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-\frac{\alpha^2}{2}} d\alpha$$

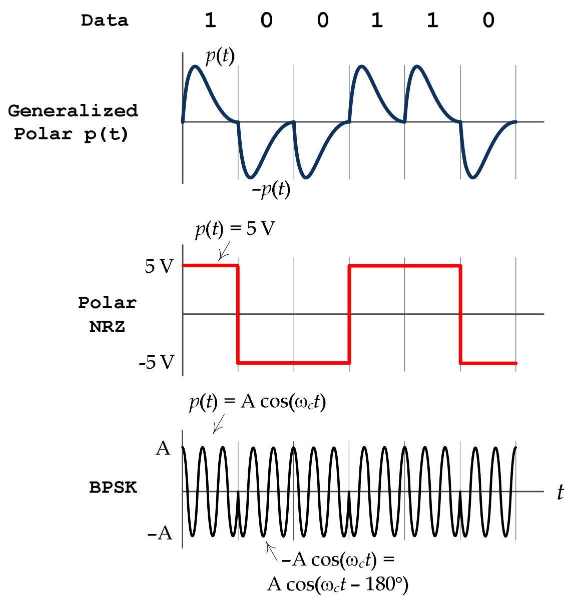


Quantile Function $Q(x)$

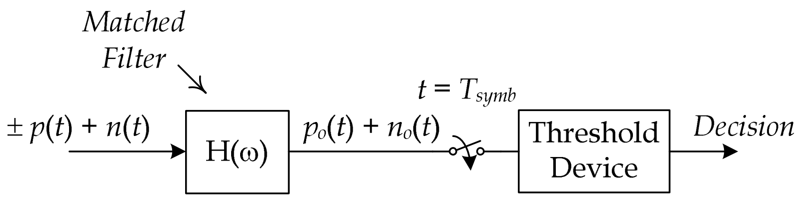
x	$Q(x)$	x	$Q(x)$	x	$Q(x)$	x	$Q(x)$
2.00	**2.28E-02**	3.00	**1.35E-03**	4.00	**3.17E-05**	5.00	**2.87E-07**
2.05	**2.02E-02**	3.05	**1.14E-03**	4.05	**2.56E-05**	5.05	**2.21E-07**
2.10	**1.79E-02**	3.10	**9.68E-04**	4.10	**2.07E-05**	5.10	**1.70E-07**
2.15	**1.58E-02**	3.15	**8.16E-04**	4.15	**1.66E-05**	5.15	**1.30E-07**
2.20	**1.39E-02**	3.20	**6.87E-04**	4.20	**1.33E-05**	5.20	**9.96E-08**
2.25	**1.22E-02**	3.25	**5.77E-04**	4.25	**1.07E-05**	5.25	**7.60E-08**
2.30	**1.07E-02**	3.30	**4.83E-04**	4.30	**8.54E-06**	5.30	**5.79E-08**
2.35	**9.39E-03**	3.35	**4.04E-04**	4.35	**6.81E-06**	5.35	**4.40E-08**
2.40	**8.20E-03**	3.40	**3.37E-04**	4.40	**5.41E-06**	5.40	**3.33E-08**
2.45	**7.14E-03**	3.45	**2.80E-04**	4.45	**4.29E-06**	5.45	**2.52E-08**
2.50	**6.21E-03**	3.50	**2.33E-04**	4.50	**3.40E-06**	5.50	**1.90E-08**
2.55	**5.39E-03**	3.55	**1.93E-04**	4.55	**2.68E-06**	5.55	**1.43E-08**
2.60	**4.66E-03**	3.60	**1.59E-04**	4.60	**2.11E-06**	5.60	**1.07E-08**
2.65	**4.02E-03**	3.65	**1.31E-04**	4.65	**1.66E-06**	5.65	**8.02E-09**
2.70	**3.47E-03**	3.70	**1.08E-04**	4.70	**1.30E-06**	5.70	**5.99E-09**
2.75	**2.98E-03**	3.75	**8.84E-05**	4.75	**1.02E-06**	5.75	**4.46E-09**
2.80	**2.56E-03**	3.80	**7.23E-05**	4.80	**7.93E-07**	5.80	**3.32E-09**
2.85	**2.19E-03**	3.85	**5.91E-05**	4.85	**6.17E-07**	5.85	**2.46E-09**
2.90	**1.87E-03**	3.90	**4.81E-05**	4.90	**4.79E-07**	5.90	**1.82E-09**
2.95	**1.59E-03**	3.95	**3.91E-05**	4.95	**3.71E-07**	5.95	**1.34E-09**

Why Quantile?





Matched Filter



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Perfromance of Digital Systems

Modulation with AWGN	**Error Probability**
ASK	$BER = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$
FSK	$BER = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$
BPSK	$BER = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$
QPSK	$BER = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$
PSK (order M)	$BER \cong \frac{2}{k} Q\left(\sqrt{2k\frac{E_b}{N_0}} \times \sin\left(\frac{\pi}{M}\right)\right)$
QAM (order M) (*Rectangular QAM*)	$P_{bc} = \frac{4}{k}\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3k}{M-1}\frac{E_b}{N_0}}\right)$ $BER = 1 - (1 - P_{bc})^2$

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Definitions

For the rest of this document, we will use the following notation:

- $\boldsymbol{M}$ = Number of possible symbols that the modulated signal can assume.
- $\boldsymbol{k}$ = the number of bits sent per transmitted symbol = **$\log_2$ (*M*).**
- $\boldsymbol{E_s}$ = *Average* energy-per-transmitted-symbol in the modulated signal (Joule).
- $\boldsymbol{E_b}$ = *Average* energy-per-transmitted-bit in the modulated signal (Joule) = $\boldsymbol{E_s/k}$.
- $S_n(\omega) = \frac{\boldsymbol{N_0}}{\boldsymbol{2}}$ = Double-sided *noise* power spectral density (*in* W/Hz = Joule).
- $\boldsymbol{T_o}$ = Bit duration.
- $\boldsymbol{T_{symb}}$ = Symbol duration = $\boldsymbol{k\ T_o}$
- **BER** = Probability of bit-error = bit error rate.

Example:
Find the BER for BPSK if we use an *optimal* detector (a matched filter). Assume the amplitude of the carrier is $A = 0.5$ V, data rate is 2 bps, and $N_0 = 2 \times 10^{-2}$ W/Hz.

Solution:
In BPSK there is one symbol per bit (i.e., a total of two symbols that the modulated signal can assume). The two symbols can be written as:

$$s_1 = A\cos(\omega_c t) \qquad\qquad s_2 = -A\cos(\omega_c t) = A\cos(\omega_c t - \pi)$$

The energy-per-symbol here is the same as the energy-per-bit and is equal for both possible symbols. Hence, its average is:

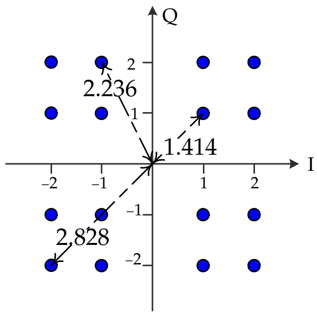
$$E_b = E_s = \left(\frac{A^2}{2}T_{symb}\right)\Pr[\mathbf{1}] + \left(\frac{A^2}{2}T_{symb}\right)\Pr[\mathbf{0}] = \frac{A^2}{2}T_{symb} = \frac{A^2}{2}T_o = \frac{A^2}{2}\frac{1}{f_0}$$

Hence,

$$\text{BER} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{A^2}{N_0 f_0}}\right) = Q\left(\sqrt{\frac{0.5^2}{2\times10^{-2}\times 2}}\right) = Q\left(\sqrt{6.25}\right) = Q(2.5)$$
$$= 6.21\times10^{-3}$$

Example

Find the BER for the 16-QAM constellation shown below if we use an *optimal* detector (a matched filter). Assume the data rate is 4 bps, and $N_0 = 5 \times 10^{-2}$ W/Hz.



Solution:
In this system there are 16 possible symbols, which we assume to be equally probable, i.e., each occurs with a probability of 1/16. Hence, the energy-per-symbol is:

$$E_s = \left(\frac{1.414^2}{2} T_{symb}\right)\left(\frac{4}{16}\right) + \left(\frac{2.236^2}{2} T_{symb}\right)\left(\frac{8}{16}\right) + \left(\frac{2.828^2}{2} T_{symb}\right)\left(\frac{4}{16}\right)$$

$$E_s = [0.25 + 1.25 + 1]T_{symb} = 2.5(T_{symb})$$

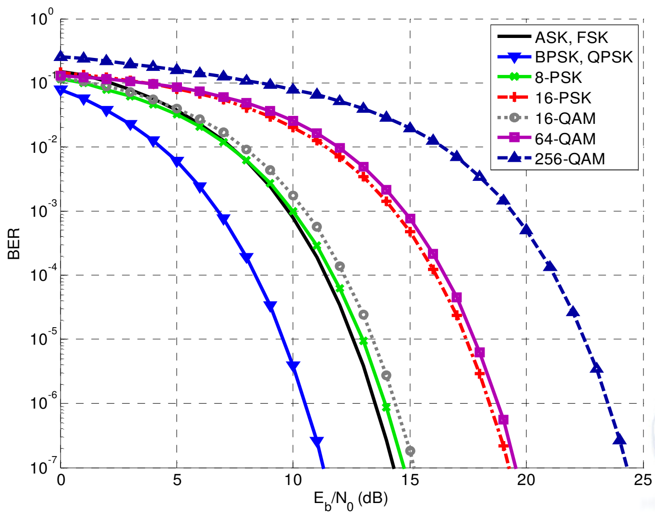
$$E_b = \frac{E_s}{k} = 2.5\left(\frac{T_{symb}}{k}\right) = 2.5(T_0) = \frac{2.5}{f_0} = \frac{2.5}{4} = 0.625 \text{ J}$$

$$P_{bc} = \frac{4}{k}\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3k}{M-1}\frac{E_b}{N_0}}\right) = \frac{4}{4}\left(1 - \frac{1}{\sqrt{16}}\right) Q\left(\sqrt{\frac{3 \times 4}{16-1} \times \frac{0.625}{0.05}}\right) = \frac{3}{4} Q(\sqrt{10})$$

$$= \frac{3}{4} Q(3.162) = \frac{3}{4} \times 8 \times 10^{-4} = 6 \times 10^{-4}$$

$$BER = 1 - (1 - P_{bc})^2 = 1 - (1 - 6 \times 10^{-4})^2 = 1.2 \times 10^{-3}$$

Comparison



Comparison

Modulation	Bandwidth	Error free Eb/No (i.e., BER < 10^{-6})
ASK	$2f_o$	13.5 dB
FSK	$2\Delta f + 2B = 2f_o(\beta + 1)$	13.5 dB
BPSK	$2 \times Baud = 2f_o$	10.5 dB
QPSK	$2 \times Baud = f_o$	10.5 dB
8-PSK	$2 \times Baud = 2f_o/3$	14 dB
16-PSK	$2 \times Baud = f_o/2$	18 dB
16-QAM	$2 \times Baud = f_o/2$	14.5 dB
64-QAM	$2 \times Baud = f_o/3$	18.5 dB
256-QAM	$2 \times Baud = f_o/4$	23.4 dB

Remember: Digital Modulation

- **Bandwidth** of the channel decides the **baud rate** (symbols per second) you can send.
- **Signal-to-noise ratio** (E_b/N_0) decides the level of modulation you can use while still maintaining a small bit error rate. In other words, it decides the number of **bits you can send per symbol**.
- Hence, the two factors together (bandwidth and SNR) decide the **total bit rate** you can achieve over any single channel.
- Shannon's Limit!

Shannon's Limit

$$C = B_{ch} \times log_2(1 + SNR)$$

- *C*: Capacity of the channel in bits/second (bps)
- B_{ch}: Channel bandwidth (units of Hz)
- *SNR*: Signal-to-Noise Ratio (unitless)(*not dB)*

Applications

- **IEEE 802.11 (Wi-Fi):** BPSK, QPSK, 16-QAM, 64-QAM and CCK (Complementary Code Keying) (CCK is an extension of QPSK).
- **IEEE 802.16 (Wi-MAX):** BPSK, QPSK, 16-QAM, and 64-QAM. It uses these modulation schemes in combination with OFDM (Orthogonal Frequency division multiplexing) (OFDM is an extension of FDM).
- **DVB (Digital Video Broadcasting):** DVB-S (for satellite broadcasting) uses QPSK or 8-PSK; DVB-C (for cable) uses 16-QAM, 32-QAM, 64-QAM, 128-QAM or 256-QAM; and DVB-T (for terrestrial television broadcasting) uses 16-QAM or 64-QAM.
- **DAB (Digital Audio Broadcasting):** DQPSK (Differential QPSK) (DQPSK is a variation of QPSK).
- **ADSL:** QAM in a scheme called DMT (Discrete Multi-Tone modulation).